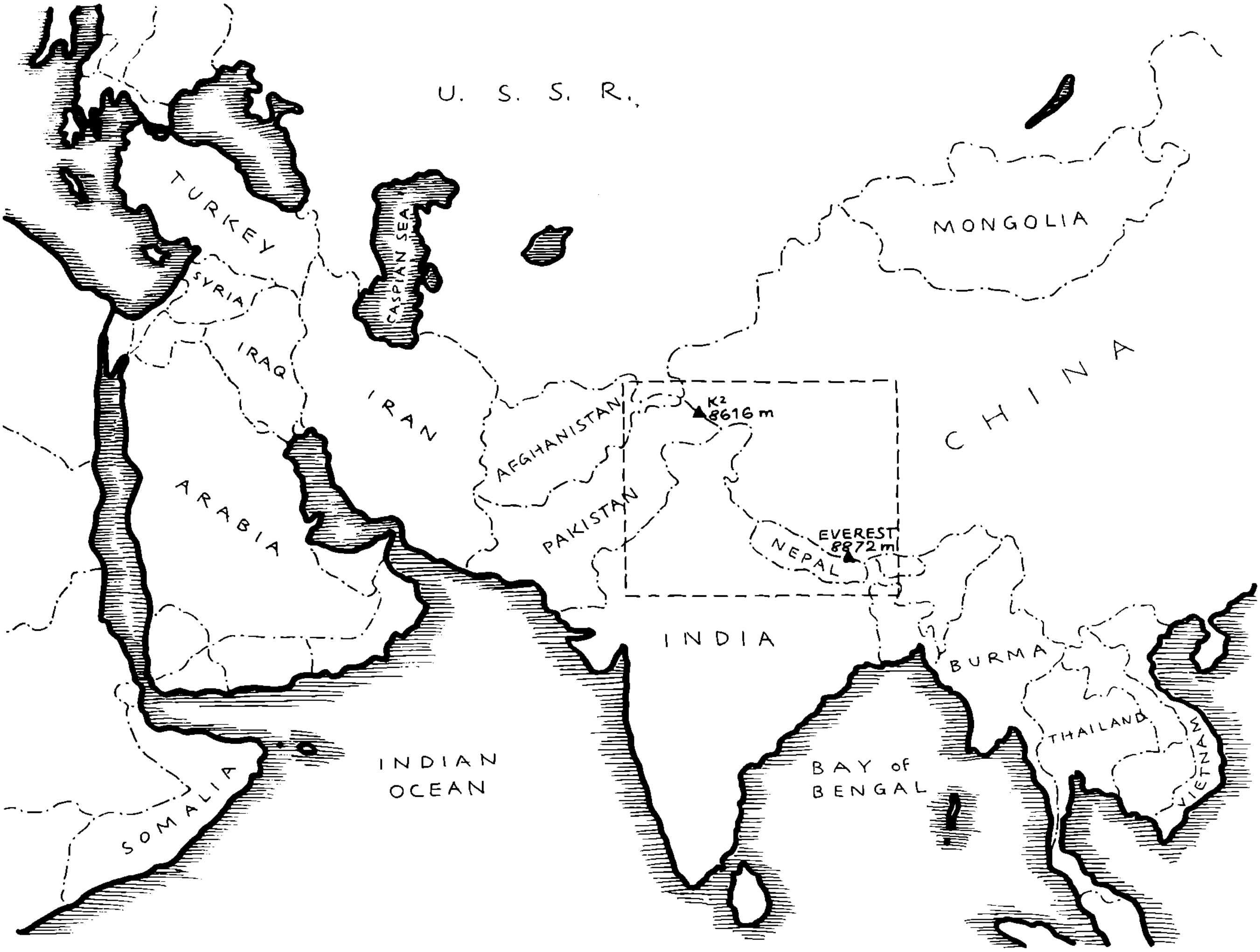
Which is the highest mountain in the World?

Report of the expedition Ev-K²-CNR to the Mt. Everest and K² 1987

by
Prof. Ardito Desio
leader of
the expedition*

*) Report presented to the Accademia Nazionale dei Lincei, Classe Scienze Fisiche, Matematiche e Naturali in the meeting of 9th January 1988.



General orientation sketch. In the area marked with a broken line – indicated in greater detail on the rear flap – Mount Everest and K^2 are marked with the heights measured by the expedition.

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Riassunto

Secondo la nuova misurazione del K^2, effettuata dal Prof. George Wallerstein, astronomo dell'Università di Washington, l'altezza della montagna sarebbe 8859 m, anzichè 8611 m, per cui risulterebbe 11 m più elevata dell'Everest. Il Prof. Ardito Desio, geologo dell'Università di Milano, ha ritenuto necessario effettuare un accertamento altimetrico usando per ambedue le montagne la stessa strumentazione. Grazie al finanziamento offerto dal Consiglio Nazionale delle Ricerche ha potuto organizzare una spedizione dotata di due strumenti GPS, forniti da una ditta privata di Padova. Durante il mese di agosto 1987 la spedizione si è recata ai piedi delle due montagne, che distano 1300 km una dall'altra, ed ha provveduto a compiere le misure.

La spedizione era composta da 9 membri diretti, per la parte scientifica, dal Prof. Alessandro Caporali, per la parte logistica dall'alpinista Agostino da Polenza.

Le misure si riferiscono all'ellissoide internazionale WGS 84 (World Geodetic System 84) e le altezze ellissoidiche del Monte Everest e K^2 risultarono rispettivamente 8833 m e 8579 m. Per convertire questi valori ad altezze ortometriche, l'altezza del geoide rispetto all'elissoide WGS 84 è stata calcolata ai due siti sulla base di un recente modello del campo gravitazionale terrestre denominato GEM-T1, derivato da NASA-Goddard Space Flight Center e che è completo fino ad ordine e grado 36. Le altezze ortometriche risultarono 8872 m e 8616 m per l'Everest e il K^2, i valori ufficiali essendo 8848 m e 8611 m. Analogamente le altezze ortometriche del Falchan Kangri (Broad Peak) e Gasherbrum IV risultarono rispettivamente 8060 m e 7929 m, mentre i valori ufficiali sono 8051 m e 7925 m.

La ripetibilità dei posizionamenti con i satelliti del Global Positioning System a intervalli di un giorno è di 20,30 m all'Everest e 7,42 m al K^2, Falchan Kangri e Gasherbrum IV. La ripetibilità delle misure con il teodolite per diversi giorni e combinazioni di punti di osservazione è 2,97 m e 0,18 m. Il ritardo di gruppo dovuto alla propagazione nella ionosfera dei segnali in banda L1 è stato modellato analiticamente. I satelliti GPS sono stati osservati a elevazioni $>20°$, pur con una diluizione geometrica di precisione (GDOP) ≤ 5.

I margini di incertezza nelle nostre stime altimetriche sono quantomeno uguali alle ripetibilità delle determinazioni delle quote dei capisaldi d'appoggio con il GPS. Due sono le fonti di possibile errore sistematico residuo: l'effetto della ionosfera e la quota locale del geoide rispetto all'ellissoide WGS 84. Mentre la calibrazione dell'effetto ionosferico potrà essere effettuata con i ricevitori prossimamente disponibili in grado di sintonizzarsi su entrambe le portanti L_1 e L_2, correzioni ortometriche con precisione migliore del metro necessitano di modelli del geoide ad alta risoluzione e misure gravimetriche locali. I valori ottenuti per le altezze complessive rappresentano un duplice miglioramento rispetto alle precedenti stime classiche: primo, superfici di riferimento definite globalmente come l'ellissoide WGS 84 o un modello globale di geoide sono usati al posto del "datum" locale e, in secondo luogo, l'effetto della rifrazione atmosferica sulle misure ottiche è stato trattato rigorosamente, usando i più recenti modelli e tecniche numeriche.

Summary

According to the new measurement of K^2 made by Professor George Wallerstein, an astronomer at the University of Washington, the height of this mountain should be 8859 m instead of 8611 m. It should therefore, be higher than Everest by 11 metres.

Professor Ardito Desio, geologist at the University of Milan, felt a control measurement necessary, using exactly the same equipment for both Everest and K^2.

Thanks to the sponsorship of the National Research Council he could organize an expedition which, equipped with two GPS systems supplied by a private organization in Padua, went up to the foot of both mountains on far from the 1300 km, in August 1987 and carried out the measurements. The expedition consisted of nine members: Professor Alessandro Caporali was in charge of the geodetic measurements and Agostino da Polenza, mountain guide, was in charge of the logistical management.

The measurements refer to the international ellipsoid WGS 84 (World Geodetic System 84) and the ellipsoidal heights of Mt. Everest and K^2 result respectively of 8833 m and 8579 m. To convert these values to orthometric heights, the ondulations of the geoid the two sites have been computed on the basis of a recent gravity field model produced by NASA-Goddard Space Flight Center, the GEM-T1 field which is complete to degree and order 36. The orthometric heights thus resulted 8872 m and 8616 m for Mt. Everest and K^2, the official values being 8848 m and 8611 m. Similarly, the orthometric heights of Falchan Kangri (Broad Peak) and Gasherbrum IV result respectively 8060 m and 7929 m, the usually given values being 8051 m and 7925 m.

The repeatibility of successive GPS fixes at one day intervals is 20.30 m for Everest and 7.42 m for K^2, Falchan Kangri and Gasherbrum IV. The repeatibility of the theodolite measurements at different days and locations transformed in terms of height variations was respectively 2.97 m and 0.18 m. The ionospheric group delay at the L1 frequency was modelled analytically. The GPS satellites were tracked at elevations higher than 20°, yet with a GDOP ≤ 5.

The uncertainty in our height estimates is at the least equal to the repeatibility of the GPS determinations of the positions of the observing sites. There are two possible residual sources of systematic error: the effect of the ionosphere and the local height of the geoid relative to the WGS 84 ellipsoid. While the calibration of the ionospheric effect can be made in the next future with receivers capable of dual band tracking, but orthometric corrections with submeter accuracy will need models of the geoid at very high resolution and local gravimetric data.

The values we have obtained for the total height represent a twofold improvement with respect to previous, classical estimates: first, globally defined surfaces such as the WGS 84 ellipsoid or a global geoid model are used in place of local datums and, secondly, the effect of atmospheric refraction on the optical measurements is treated rigorously, using recent models and numerical techniques.

A. HISTORY OF THE Ev-K^2-CNR EXPEDITION

1. Introduction

On March 7th 1987, the "New York Times" reported that, according to measurements made by Professor George Wallerstein, an astronomer at University of Washington, the highest mountain in the world was not the Everest, but K^2. The height obtained by the American scientist by means of the most modern equipment, which use signals trasmitted by a few man made satellites, turned out, in fact, 8859 metres, which is 11 metres in excess compared to the traditional height of Everest (8848 m). On March 8th also radio, television networks, and main Italian newspapers reported the astonishing news.

It was not the first time that people have tried to upset the supremacy of Everest among the highest mountains in the world. Althought Wallerstein had been very cautious in expressing the results of his measurements, media had inflated them.

I remember that in the year 1930 an American botanist and explorer — Joseph Rock — announced that in China, in Sichuan, there was a mountain which was 9250 m high, i.e. higher than Everest by 400 m. That mountain was called Minya Konka (Gangga Shan). But the height of that mountain, according to Chinese measurements, was later found to be 7550 m, i.e. 1298 m less than Everest.

Later on, Rock pointed out that there was another mountain in China, east from Kun Lun, which was higher than 9000 m: the Anye Machin. In 1949 Leonard Clark, another adventurous explorer, measured its height with traditional methods; the height of that mountain resulted 9041 m, i.e. higher than Everest by 193 m. But in the year 1970 the Chinese measured it again and with greater accuracy. After those measurements the mountain was found to be only 6282 m high, i.e. lower than Everest by as much as 2556 m. Everest kept therefore its supremacy. Sixteen years later, the news that it was challenged by K^2 were reported. But before accepting that fact it was obviously necessary to measure both Everest and K^2 with the same equipments and it is on this basis that I decided to start getting contact in order to make a new measurement possible.

In the course of a chance meeting on April 11th with professor Luigi Bernardi, President of the CNR (Consiglio Nazionale delle Ricerche), I suggested sponsoring an expedition to remeasure the heights of K^2 and Everest in situ, using identical and the most modern equipment. This suggestion met with instant approval.

I got down straight away to making an initial rough draft for the organization of the expedition.

Little more than a week later, I received a telephone call from Agostino da Polenza in Bergamo. This mountaineer was the first Italian to scale the north (Chinese) wall of K^2, during the Santon Expedition in 1983; he runs a kind of mountaineering tourist bureau which organizes climbing expeditions to peaks of 8000 m. Agostino wanted me to shed some light on the possibility of measuring the height of K^2. I then told him about my project, and the following morning he dropped in to Milan to meet me.

This was the beginning of a fruitful partnership between the two of us, and it was at this point that preparations for the expedition started getting underway in earnest. The speed was essential, for it was imperative not to take up too much of the expedition members'time, and to keep costs down to a minimum.

2. The Organization of the expedition

At this point I should remind you that the common altimetric figures of 8611 m for K^2 and 8848 m for Everest were obtained with traditional equipment and measuring systems about half-way through the last century. These measurements were carried out by the Survey of India (the Indian Topographical Service) and at the time, more precise calculations than these could not have been made.

So, on April 27th I delivered a memorandum to the President of the CNR (Consiglio Nazionale delle Ricerche), which contained a brief plan of the expedition drawn up in consultation with Agostino da Polenza. The memorandum was approved by the Presidential Council of the CNR, and then by the Administrative Board, the two decision-making bodies of the CNR, and they made the necessary funds available.

In order to carry out the measurements, it was necessary to get hold of equipment of a newer generation than that adopted by Wallerstein, equipment known by initials GPS (Global Positioning System). In the first istance, I got in touch with the Istituto Geografico Militare Italiano who seemed willing to help out, but I was subsequently informed that for the moment they did not have this equipment at their disposal.

This news meant that I urgently needed to find another organization who might be in the position to supply us with the necessary equipment and personnel to perform the measurements. This was by no means an easy task, since, whatever door I knocked at, I always came away with a negative response. This was essentially due to the fact that these organizations did not actually have GPS available. A whole month went by in this fashion.

Finally, at the beginning of June, I managed to find a private organization in Padua. In that time this was the only body in Italy that not only owned two operational GPS systems, but also had personnel trained to use them. Negotiations were swiftly concluded; not, however, without considerably increasing our financial commitment. While the Society's technicians, under the direction of Professor Alessandro Caporali — a lecturer at the University of Padua — went off to the Dolomites to practice with the equipment, Agostino da Polenza took advantage of the fact that at that time he had the job of accompanying a mountain-climbing expedition to Nanga Parbat (8114 m) in Pakistan to get in touch with the authorities of that country to request the use of two military helicopters in service on the borders, in order to get quickly to the operative base-camp of K^2 on the Baltoro glacier. Bear in mind that the border between Pakistan and Indian, which is still garrisoned by troops, runs along the crest of the mountain. In making these approaches, we were rendered assistance by General Omar Ali Mirza, President of the Pakistan Alpine Club, who was taken on as a member of the expedition.

As far as the measurement of Everest was concerned, the logistical problem appeared to be more strightforward than had been anticipated. This was due to the fact that the mountaineer and guide Renato Moro, who had just returned from leading an expedition to Everest, informed me that the base-camp, at a height of 5300 m on the Tibetan slope near the monastery of Rongbuk, could actually be reached by motor vehicles, and he offered to accompany the expedition himself. Therefore, in view of the lesser difficulties involved in carring out this operation, I decided to give precedence to the measuring of Everest.

The expedition party, consisting entirely of unpaid volunteers, was made up as follows:

— Prof. Ardito Desio, in overall charge of expedition;
— Prof. Alessandro Caporali, in charge of geodetic measurements;
— Engineer Lionello Lavarini, assistant to Caporali;
— Engineer Claudio Pigato, assistant to Caporali;
— Dr. Attilio Bernini, doctor;
— Mino Damato, journalist;
— Agostino da Polenza, mountain guide, in charge of logistical management;
— Kurt Diemberger, cine-photographer;
— Renato Moro, mountaineer, in charge of logistical management for Everest along da Polenza;
— Soro Dorotei, alpine guide, assistant to da Polenza for K^2.

The CNR was represented by Ernesto Brambati, accountant who was to handle administrative affairs.

As for myself, I intended to step in only if it should turn out that my presence was required, as did in fact happen during the course of operations in Pakistan.

3. The operation of the expedition on Everest and K^2

About at half of July the organization of the expedition was at the end, and on July 28th the members with Agostino da Polenza, as leader, accompained by Renato Moro, set off by air for Katmandu, the capital of Nepal.

After two weeks no further news I received from them, until on the 10th of August, out of the blue, came the long-awaited telephone cal informing me that the measurement of the height of Everest had been completed in magnificent weather conditions, and that all the members of the expedition were already making preparations to leave for Pakistan.

I experienced a considerable sense of relief on hearing this news, after the worries that had put me under pressure so much of the time during the preparatory phase of the expedition. At the same time, I clearly felt that the trust I had placed in my men, who were engaged in an operation which was far from easy, had been more than merited. This assured me that the Pakistan operations, although accompanied by even greater difficulties to be surmounted, would succeed in completing the scientific programme within the projected time of about one month. And, so, on August 15th the expedition party disembarked at Islamabad, the capital of Pakistan, and go ready to start off again for Karakorum. That evening, however, Agostino da Polenza rang to inform me that my presence as leader of the expedition was required, to resolve certain logistical problems. However, I could not leave Milan then because my wife was indisposed, so I sent a telegram to say that I would arrive the following week. So it was that, with the help also of General Mirza, the expedition party, which now also included the mountaineer Soro Dorotei, left Islamabad and drove to the oasis of Skardu.

There we had some difficulty in setting off again in the military helicopters for the Concordia base-camp on the Baltoro glacier on account of bad weather. In the end, however, one of the elicopters managed to transport just the operators to a point near Urdukas (the base-camp of the Italian expedition 1929). From there they continued on foot to Concordia. Here once again luck was on our side since, thanks to the magnificent weather. Caporali's team was able to complete the measurement not only of K^2 but also of Falchan Kangri (Broad Peak) and Gasherbrum IV in only four days. After this, the whole expedition party returned to Skardu and from there, on August 29th, they transferred to Islamabad, where myself had already been for a few days.

That same day I sent a telegram to the President of the CNR announcing that the expedition party safely returned to Islamabad and that the planned research was successfully completed.

That evening, the President of Pakistan, General Mohammad Zia-Ul-Haq, who had been in Karachi on State business for some time, returned to Rawalpindi and received the members of the expedition party, including General Mirza, in his residence. After listening to my account with great interest, the President offered some words of congratulation on the outstanding achievement of the expedition, which "provided a further significant contribution by Italy to scientific research in Pakistan". Before taking leave of us he presented gifts to all the members of the expedition.

That night, the entire expedition party left Pakistan with their burdensome luggage and arrived in Milan late in the afternoon of August 30th. At Linate airport Professor Rossi Bernardi, President of the Consiglio Nazionale delle Ricerche, was waiting to greet the party; he congratuled the travellers returning from their long journey on fully attaining the goals of the expedition in record time, that is in the space of about one month, using the most modern equipment to measure the heights of Everest, the highest mountain in the Himalayas, and K^2, the highest mountain in the Karakorum.

B. THE MEASUREMENT OF THE HEIGHT OF K^2 AND EVEREST*

1. Introduction

Mt. Everest is credited to be higher than K^2 by approximately 237 m, the usually quoted heights being respectively 8848 m and 8611 m. The value for Everest is deduced from the 1 : 50,000 map of the Kumbu Himal based on trigonometric starting points of the Survey India and field work by E. Schneider and collaborators from 1955 to 1963. This is the value most commonly found in the maps and atlases, certainly not the only one. Several other values are reported. For instance, the value officially quoted by the Survey of India is 8840 m and was determined by Waugh with coefficents of refraction varying from 0.07 to 0.08 from stations in the plains and measurements from 1849 to 1902. The value of K^2 was deduced by Montgomerie with coefficents of refraction varying from 0.04 to 0.05 and measurements from 1857 to 1859 from Mt. Haramukh (4877 m) at a distance of 212 km. (Burrard and Hayden, 1933).

These values are intended to furnish a mean of identification and, as such, should not be altered frequently or without a good reason. Still, from the scientific viewpoint, it is certainly open to debate how the raw trigonometric data have been processed and which distance these numbers physically represent. As it appears already in the original reports of the Survey of India and is now well visible from global geoid maps obtained by satellite techniques and surface gravimetric data, in these regions the structure of the equipotential surfaces relative to a reference ellipsoid is very complicated. The consequent, relevant deviations from the vertical make the interpretation of the triangulation measurements of mountain heights taken from large distances not immediate nor obvious, also because of the competing effects of atmospheric refraction. In comparing the height of a mountain at different epochs the question arises as whether the range it belongs may be rising or subsiding. This has been pointed out to be relevant to the Great Himalaya range, where earthquakes which occurred after the official measurements were taken are suspected to have produced changes in height of as much as several tens of meters.

In comparing the heights of widely separated mountains, such as Everest and K^2 (1300 km), the most difficult question is how to ensure that a consistent reference surface is used as zero for the height. This question can, in fact, hardly be satisfactorily answered without some major technological development which permits to limit the uncertainties related to the definition of a common reference datum.

In the past thirty years, since the launch of Sputnik in the occasion of the International Geophysical Year, the development of space techniques and their application to precision geodetic measurements have brought to a major breakthrough in the determination of absolute positions as well as relative distances. The orbits of artificial satellites can now be determined with meter accuracy. With the satellites forming the Global Positioning System (GPS) of the United States, the tracking station reduces to a small receiving antenna mounted on a tripod and a light, self

* With the collaboration of Prof. Alessandro Caporali, leader of the survey team.

contained receiver box. The accuracy in positioning is guaranteed by the broadband structure of the Pseudo Random Noise (PRN) codes modulated on high frequency carriers on the one hand, and, on the other hand, the simultaneous availability, in most areas of the world, of up to five satellites during certain, well predictable periods of the day. The position on the center-of-phase of the antenna is determined relative to the geocentric international ellipsoid WGS 84 (World Geodetic System 84) and is thus independent of the definition of the the local datum and mean sea level. For these reasons, satellite techniques become very attractive in addressing the problem of measuring and comparing mountain heights using a consistent reference surface.

2. The expedition's technical assignments

As mentioned at the beginning of this report, the main objective of the Ev-K^2-CNR 1987 expedition was first and foremost to establish whether K^2 was in fact higher than Everest, as claimed in the international press on the basis of Wallerstein's measurements on K^2. This was the primary question and I believe it can now be said to have been settled once and for all.

The second objective was to find out how much higher Everest is than K^2 in terms of size. From measurements we took the answer would appear to be that the difference is 250 m, i.e. not greatly different from the traditional measurement of a hundred years ago of 237 m.

The third objective was to establish the current height above sea level of Everest and K^2, a problem which is closely linked to the previous one. This third objective was only achieved with approximate measurements owing to the limited time available for geodetic operations at the locations for the reasons explained in the first part, and owing to the need to supply answers in a fairly short space of time to the numerous and pressing requests I received from all over the world. The results given are those obtained from the instruments used, i.e. a preliminary elaboration of the data collated rather than a final version. I should state at this point that another expedition will in fact be needed to give full definitive answers to these third objectives.

3. The result of the measurements

The instrumentation included a precision electronic theodolite Wild T2000 coupled with an Electronic Distance Meter (EDM) Wild TI5S, and two GPS receivers Wild Magnavox WM 101. The project of the geodetic network at K^2 was made on the basis of the map of the Italian Expedition to K^2 (1954) surveyed by an officer of the Istituto Geografico Militare Italiano (Francesco Lombardi) and of photographs of the Italian 1953-4 and 1983 expeditions.

Everest was observed from the Tibetan side at the height of 5200 m using a network extending in the north-south direction from the Rongbuck monastery to the beginning on the Rongbuck glacier and, in the east-west direction, covering the full width on the valley of the Rong River (fig. 1). Table 1 summarizes the mean angle and distance observations, taken from the 4th to the 7th of August. The GPS antennas occupied permanently positions 1 and 4.

K^2 was observed from Concordia at about 4600 m on the Baltoro glacier, at three well spaced and very favourable sites which made unnecessary the establishement of a larger network (fig. 2). Table 2 summarizes mean angles and distances, observed from the 21th to the 23th of August. Only one GPS equipment could be transported and it occupied permanently position 1.

Table 1 and 2 also give the compensated trigonometric heights of Mt. Everest and K^2 respectively relative to the corresponding GPS sites 1, after a distance scaling of 133 parts per million (ppm) for Everest, and 113 ppm for K^2, as mean correction terms specified by the manufacturer of the EDM for the relevant meteorological conditions.

Tables 3 and 4 give the ellipsoidal heights of the GPS sites 1 at Everest and K^2 respectively. These heights have been computed by processing the data recorded on the cassettes, by means of the PoPS (Post Processing Software) package.

From the data contained in Tables 1 through 4 the ellipsoidal height "h" of Everest and K^2 can be computed as follows:

$$
\begin{aligned}
h &= H(\mathrm{GPS1}) + D^2/2r^\circ + h(\mathrm{TRIG}) \\
&= 8833 \text{ m for Everest} \\
&= 8579 \text{ m for } K^2
\end{aligned}
$$

Here H(GPS1) is the ellipsoidal height of the GPS site 1, given by tables 3 and 4; h(TRIG) is the trigonometric height of the summit, given by tables 1 and 2; $D^2/2r^\circ$ is the "curvature correction", D being the topographic distance-along the local tangent plane to GPS site 1-from the GPS site 1 to the projection of the plane tangent to the GPS site 1 from the local sphere of radius r°.

The relevant values are:

$$
\begin{aligned}
D^2/2r^\circ &= 50 \text{ m for Everest} \\
&= 19 \text{ m for } K^2
\end{aligned}
$$

The accurate computation of orthometric heights of the Everest and K^2 requires the knowledge of a set of coefficents which represent, in the sense of an expansion of the geopotential into spherical harmonics, the equipotential surface of the earth, as well as information on the gravity anomaly and on average topographic heights in the area. Global geoids are available from satellite and gravimetric data but, unfortunately, they all suffer from lack of satellite tracking data and terrestrial gravity measurements in these regions. Local datums exist, but we are not aware of precise transformation coefficients. The most recent global gravity field model computed by NASA Goddard Space Flight Center, GEM-T1 (Goddard Earth Model T1) is complete to degree and order 36. Recent studies (Rapp, 1987) indicate that the model, when compared to 5° equal area anomalies, has on accuracy of $\pm$ 4 mgal. The relevant geoidal ondulations are:

$$
\begin{aligned}
N &= -39 \text{ m for Everest} \\
N &= -37 \text{ m for } K^2.
\end{aligned}
$$

For comparison, other global models have been used as Rapp 1980 and others, complete to degree and order 180, which give a somewhat smaller difference geoid ellipsoid. The orthometric heights of the two mountains are respectively:

$$
\begin{aligned}
h &= 8872 \text{ m for Mt. Everest} \\
h &= 8616 \text{ m for } K^2.
\end{aligned}
$$

From table 2 the total ellipsoidal and orthometric heights of Falchan Kangri (Broad Peak) and Gasherbrum IV can also be computed using the same method as above. The orthometric heights are:

$h = 8060$ m for Falchan Kangri (Broad Peak)
$h = 7929$ m for Gasherbrum IV.

4. Error analysis

4.1. Accuracy of the ground survey

4.1.1. Resolution of the instruments

The theodolite Wild T2000 has a digital readout of both zenith and azimuth angles with a resolution of 0.0001 centesimal degrees. It also has a built-in system of vertical compensation which automatically corrects for leveling drifts the zenith angle and permits to compute a corresponding correction in azimuth. Thus, by complementing each observation in a given direction with measurements of the vertical offset in the two orthogonal directions and applying the relevant azimuthal correction formula to the raw azimuth data, thermal and other drifts can be accounted for and the nominal resolution can be consistently maintained throughout the entire observing session.

~~As to the di~~stances, the measurements with the ~~EDM showed repeatibilities~~ within few millimetres over distances of up to 5 km, and were scaled according to the specifications of the manufacturer, as mentioned earlier. At Mt. Everest the 1-4 baseline (see table 1) was also measured by GPS in translocation mode using double differenced phase data. As the phase ambiguities were successfully resolved for all pairs of satellites, the GPS measurement of the slope distance is normally fairly accurate. Table 5 summarizes the results of the translocation analysis and also gives, for comparison, the unscaled EDM distance. The scale difference between the GPS and EDM estimates is thus 146 ppm, certainly consistent with our adopted 133 ppm in consideration of offsets between the optical centers of the EDM and the mirrors an the one hand, and the (varying) centers-of-phase of the antennas on the other hand.

Besides the internal accuracy of the instruments, there are three major sources of error in the ground survey: collimation of the summit, atmospheric refraction and deflection of the vertical. These are discussed in the following three subsections.

4.1.2. Collimation of the summit

A subjective bias in the collimation of the summit is an important potential source of systematic error. Our team included Italian mountaineers who had climbed both mountains and were therefore most helpful in collimating the right spot. In addition, two (occasionally three) surveyors made independent measurements with the theodolite. The sets of angles measured by each surveyor were compared against each other to identify biases. The result of this test proved negative.

4.1.3. *Atmospheric refraction*

The atmospheric refraction has always been a most controversial issue. Changes in published heights of these mountains have often reflected different points of view and understanding of how to increase the observed zenith angle to account for the bending of light rays propagating in a atmosphere with changing index of refraction. Our own treatment is summarized with reference to fig. 4. Snell's law in a spherically stratified medium states the constancy along the ray path of the product of the local index of refraction *n(r)*, the geocentric radius *r* and the sine of the local zenith angle δ:

$$n(r)r \sin(\sigma) = \text{constant}.$$

At the observation point, where the theodolite is located, $r = r^\circ$ (known from GPS), $\delta = \delta^\circ = z^\circ\pi/200$ rad, is the observed zenith angle of the summit. We introduce the constant k:

$$n(r^o)\ r^o \sin(\sigma^o) = k,$$

where $n(r^o)$ is computed from pressure, temperature and humidity data using formulae and models given in the appendix.

The differential equation of a ray in a spherically symmetric medium is (Born and Wolf, 1959; Mueller, 1969)

$$r\frac{d\theta}{dr} = \frac{k}{\sqrt{(n^2r^2 - k^2)}}$$

where θ is the center angle (see fig. 4). Indicating by $h = r - r^o$ the height over the local sphere of radius r^o, this differential equation is converted to a integro-differential equation where the height $h1$ of the summit is the only unknown:

$$\frac{\theta}{k} = \int_0^{h1} \frac{dh}{(r^o+h)\sqrt{n^2\,(r^o+h)^2 - k^2}}$$

Note that

$$\theta = D[1-1/2\ (D/r^o)^2]/r^o,$$

where D is the topografic distance introduced earlier and is known, to good approximation, independently of refraction. h1 is determined by successive trial numerical integrations of the right hand side of eq. (1) until eq.(1) is satisfied with negligible numerical error.

If $z^o + \delta z$ is the zenith angle we would observe in the absence of atmosphere, then the required atmospheric correction to be added to the raw zenith angle z^o is

$$\delta z = \arctan\,[D/(h1 - D^2/2r^o)] - z^o$$

This quantity was computed for each observeation session of typically 15 minutes on the basis of pressure, temperature and humidity data at the instrument, azimuth angles (which can be assumed independent of refraction) and GPS estimates of r^o. The value of z given in Table 1 and 2 are averages of all sessions after having applied the relevant δz to the mean z^o of each session.

The treatment of refraction outlined above was preferred to others, often used by surveyors and astronomers. Geodesy and topography textbooks (see, e.g., Torge, 1980; Bartorelli, 1986) give the formula

$$\delta z = D/2r^o$$

where k (not to be confused with our k) is chosen between 0.06 and 0.10. The choice is to some extent subjective and the ensuing, non negligible uncertainty has, in fact, been subject of considerable debate.

The Astronomical Almanac proposes for the reduction of astronomic observations

$$\delta z = 0^o.00452\ (P/T)\tan(z)$$

with P = total air pressure in millibar and T = air temperature in °K. The accuracy is 0'.1 above 15° elevation. The simple dependence on the tangent of the zenith angle implies that the atmosphere is treated as a thin lens with constant refraction index. If one also considers the small elevation angles we work with and the implicit assumption of infinite distance of the target, then one concludes that this expression is of limited applicability to our case.

4.1.4. *Deflection of the vertical*

As to the deflection of the vertical, we have no detailed gravimetric data available and therefore we have applied no corrections of this type to the zenith angles. Some idea on its value can be deduced from the table 1 and 5 in the case of Mt. Everest, were we did combined theodolite-EDM and differential GPS measurements. The former give differences in orthometric heights and the latter give differences in ellipsoidal heights. Thus the difference in the two types of relative height, the so-called difference of geopotential numbers, gives at least an upper limit on the deflection of the vertical integrated over the 1-4 baseline. This line was chosen closest to the north-south direction, where in fact the maximum vertical deflection is expected. Table 1 gives a geoidal height difference of 161.21 m between site 4 and site 1; table 5 gives an ellipsoidal height difference of 160.94 m. Because the baseline is 5961 m long, the deflection of the vertical should be smaller than 0.003 degrees centesimal, and is therefore comparable with other error sources. This value is confirmed by predictions made on the basis of GEM-T1, RAPP 1981 and other geoidal models. An approach based on a more detailed field and local gravimetric data is given by Engelis et al. (1985).

We are aware that in such an area in reality the deflection of vertical may amount to larger values. Unfortunately, time limitations have prevented us from realising a direct estimate by astronomical techniques. We hope to do it in the next future.

Table 6 summarizes the theoretical error budget of the ground survey measurements. An upper limit to the trigonometric height error resulting from network geometry is estimated by adding the root sum square (r.s.s.) error to the elevation or azimuth angles of the weakest baselines in table 1 and 2 (i.e. those forming the smallest angles at the summit). The conclusion is that 10 m can be considered a worst case, theoretical upper limit for the error in the trigonometric heights. It is important to note that this figure is, in fact, larger of the actual repeatibility of our compensated trigonometric height estimates for all sites and for both mountains, thereby implying that our measurements are scattered on a band narrower than the theoretical error budget.

4.2. Accuracy of the GPS Measurements

The geocentric location of the center-of-phase of the GPS antenna is, at each instant, determined by passive multilateration to four or more GPS satellites. Multilateration is done by matching the PRN code modulated on the carrier with a replica generated within the receiver and detecting the difference between the epoch (in the GPS time scale) of transmission and the epoch (in the receiver time scale) of reception of the wave packet. By tracking at least four satellites at once, the instantaneous offset of the crystal oscillator internal to the receiver relative to the time kept by the atomic clocks on board the GPS satellites — which is a known function of the Universal Coordinated Time UTC — is determined simultaneously with the three spatial coordinates of the antenna. Sequential Kalman filtering provides real time "quik look" update of the coordinates on the receiver display, as the satellites move across the sky. Computer post processing of the data stored in cassettes provide a more reliable solution.

Each satellite continuously boradcasts navigation signals at two carrier frequencies: L1=1575.42 MHz and L2=1227.60 MHz. The L1 wave is BPSK (Binary Phase Shift Keying) modulated by two PRN codes: a P code ranging signal, with bandwith 10.23 MHz and a C/A code ranging signal, with bandwidth 1.023 MHz. The two sequences are in quadrature and are coherently synthesized by the same oscillator working at 10.23 MHz. On the L1 carrier, data at 50 BPS (bits per second) are also modulated to provide satellite ephemeris and clock bias (relative to UTC) information. The L2 carrier is modulated only by the P code.

Our WM 101 receivers can, at present, collect C/A coded data at the L1 frequency. If we adopt the "rule of thumb" that the observation resolution is about 1% of the signal wavelength, then the nominal resolution with the C/A coded pseudoranges (wavelength=300 m) is 3 m (Wells, 1986). The overall system accuracy in absolute positioning is, however, determined by other error sources. The most important are: receiver noise, multipath, ionospheric and tropospheric effects, satellite ephemeris errors, satellite clock errors, earth orientation errors and satellite geometry (King et al. 1985). Of these, the most unpredictable are certainly the ionospheric effects. They produce the largest bias when the satellite is at the horizon, near midday and in periods of maximum sunspot activity. The Post Processing Software PoPS models the ionospheric zenith delay with a time-dependent scale factor consisting of a constant plus, in the interval 8-20 of local time, a cosine bell centered at 2 pm. The scale constants are respectively 10^{15} and $4 * 10^{15}$ electrons/m^2. This delay is mapped to zenith angles $\leq 75°$ by a $1/\cos(z)$ function (Bauersima, 1983). A typical error size is 30 m which should then be considered with the due caveats.

We summarize in table 7 the theoretical GPS error budget. It indicates a r.s.s. theoretical model error of 42.7 m, which in consistent with our determinations of the GPS sites (table 3 and 4). The satellite geometry defines the Geometric Dilution of Precision (GDOP) and the scale factor which should be applied to the nominal resolution to calculate the r.s.s. user's position variance. As we have consistently scheduled our observing sessions with a GDOP ≤ 5, then the geometric error should be ≤ 15 m, if the nominal resolution is 3 m with the C/A code.

The coordinates of the GPS stations have been determined on the basis of all available measurements, though some of them may be doubtful. We have deemed a correct attitude to keep all measurements rather than rjecting some of them on the basis of such a small population.

5. Conclusion

A comparison with older measurements (Burrard and Hayden, 1933) shows that 8872 m, the orthometric height of Everest we have determined, is closer to that (8882 m) measured in 1905 from the hills using a coefficient of refraction of 0.05, or to the value 8852 m deduced in 1931 by dr. De Graaf Hunter using a refined model of refraction than to the value (8848 m) usually given in the maps or the value (8840 m) determined by Waugh. This latter value was felt too low already in the cited paper by the Survey of India.

The uncertainty (1σ) of our value for Everest is somewhere between ± 20 and ± 30 m, and is due primarily to the performance of the GPS and to the local profile of the geoid. The orthometric height determined for K^2, 8616 m, is slightly higher than that, 8610,6 m, measured by col. Montgomerie, yet not enough to confirm Wallerstein's preliminary observation. The estimated uncertainty is $\pm$ 7 m and $\pm$ 17 m. Because the height increase is also marginal for the nearby mountains, the officially quoted heights can be considered as firm.

Although our heights cannot be assumed as definitive, it is symptomatic that all the figures we obtained show greater heights than the traditional ones with differences ranging from 4 m for Gasherbrum IV, to 24 m for Everest.

If we assume provisorily that there have been no errors in the old and recent measurements, the area of K^2 must have risen in the last century at a rate of 2.8 cm a year, and that of Everest at a rate of 17,5 cm. In the light of what is currently known, these figures seem to be excessive, at least with regard to Everest.

How can we explain this problem?

The two most plausible explanations are either that there has been a sudden elevation of the mountains in the last century, or that the measurements were imprecise.

The most logical conclusion seems to be to attribute the difference partly to one factor, partly to the other.

At this point, I should like to mention an experience concerning the Everest area.

Some years ago, towards the end of an excursion across southern Tibet in the company of some Chinese colleagues, not far below the Yagru Shohn pass (5122 m), at a height of 4950 m, some fragments of bone were pointed out to me. They occurred within a sandy layer of lacustrine origin, turned up in clearing the road.

They were in fact fossil bones of *Hipparion*, an ancestor of the horse, of which remains have also been found on the southern slope of the Himalayas only about a thousand metres above sea level.

What does all this means? It means that in the last two millions years, this area has risen 4000 m at a rate of about 2 mm a year. The figure seems to be significant, but it is too small if we compare it with the figures given for K^2 and particularly for Everest.

The problem may be solved with further investigations, particularly in the Everest area, where I hope to be able of organising in the next future further expeditions also devoted to tackling other problems to geodetic, geophysic and geologic interest in connerction with those of my previous expeditions in the Karakorum and Hindu Kush mountain ranges.

Acknowledgement

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Tables

baseline i-j	topographic distance (m)	θ_i(°)	θ_j(°)	z_i(°)	z_j(°)	δHij (m)	h (TRIG) (m)
1-2	2275.734	10.1940	188.8021	90.5280	86.6125	−5.656	3787.684
1-3	2039.568	18.4460	179.9660	90.5280	90.0118	−108.308	3796.335
1-3b	2450.107	29.4796	167.5067	90.5280	90.4135	−290.913	3793.726
1-4	5958.699	3.6488	195.2286	90.5280	88.1246	−161.207	3795.251
2-3	989.660	118.0682	79.3399	89.6125	90.0188	−102.652	3793.424
2-3b	1458.314	101.3776	94.6048	89.6125	90.4135	−285.257	3792.590
3-4	4099.860	12.8852	184.4043	90.0188	88.1246	−52.899	3796.178
						mean=	3793.598
						mean absolute error	2.028

Table 1: mean angle and distance measurements at Mt.Everest. Angles are in degrees centesimal and are corrected for refraction; heights involving points other than point 1 are referred to point 1 by applying the appropriate height difference and curvature corrections. The uncertainty is defined by the mean abosolute error. This is the sum of the absolute values of the residuals relative to the mean, divided by the number of obserbvations. A larger date set, would allow a statistically more significant extimate of the accuracy. This holds for the entire set of measurements.

baseline i-j	topographic distance (m)	θ_i(°)	θ_j(°)	z_i(°)	z_j(°)	δHij (m)	h (TRIG) (m)
1-2	1388.938	106.5141	87.8903	83.9184	84.2174	−25.986	4010.968
1-3	2531.398	125.3620	65.7017	83.9184	85.2002	−57.696	4011.167
						mean=	4011.067
						mean absolute error	0.100
Falchan Kangri (Broad Pk.):							
1-2	1388.938	69.0946	122.1198	77.7160	76.5874	−25.986	3467.554
1-3	2531.398	87.9500	94.9151	77.7160	77.7542	−57.696	3465.737
						mean=	3466.646
						mean absolute error	0.909
Gasherbrum IV:							
1-3	2531.398	40.5335	147.6611	79.6282	75.6899	−57.696	3334.926

Table 2: angle and distance measurements at K^2, Falchan Kangri (Broad Peak) and Gasherbrum IV (which was unvisible from site 2). Symbols have the same meaning as in table 1.

Antenna Nord: GPS site 1

date	duration (hh:mm)	ellipsoidal height(m)	latitude North	longitude West	note
4/8	1:40	4961.54	28°11'51".184	86°49'27".034	
5/8	1:10	4991.07	28°11'51".508	86°49'26".591	
6/8	2:05	5013.01	28°11'51".612	86°49'26".288	(1)
7/8	1:07	4991.00	28°11'51".850	86°49'26".972	
	mean=	4989.16	28°11'51".539	86°49'26".721	
	rms=	21.12	0".276	0".349	

Antenna Sud: GPS site 4

date	duration (hh:mm)	ellipsoidal height(m)	latitude North	longitude East	note
5/8	2:19	5123.42	28°08'58".276	86°51'04".268	
6/8	2:30	5161.12	28°08'58".394	86°51'04".266	
7/8	2:20	5145.23	28°08'58".427	86°51'04".090	
	mean=	5143.26	28°08'58".366	86°51'04".208	
	rms=	18.93	0".079	0".102	

mean height of GPS site 1 with heights also from site 4:4984.24
geocentric radius of GPS site 1:6371257 m
note: (1) noisy data and heavy storm

Table 3: results of point positioning post-processing of Mt. Everest GPS data

Concordia: GPS site 1

date	duration (hh:mm)	ellipsoidal height(m)	latitude North	longitude West	note
21/8	2:08	4540.13	35°44'29".587	76°30'25".736	(1)
22/8	2:22	4553.70	35°44'29".522	76°30'25".186	
23/8	1:21	4552.12	35°44'29".886	76°30'26".600	
	mean=	4548.65	35°44'29".665	76°38'25".534	
	rms=	7.42	0".194	0".300	

geocentric radius of GPS site 1: 6375855 m
(1) satellite 3 deleted from the solution; noise data from satellite 11,12,13.

Table 4: results of point positioning post-processing of K^2 GPS data

site	ellipsoidal	latitude North	longitude West height(m)	notes
antenna sud	5152.623	28°08'58".49163	86°51'04".03083	
antenna nord	4991.686	28°11'51".62279	86°49'26".72881	fixed
slope distance:		5960.96 m		
slope distance with EDM(unscaled): 5960.09 m				

Table 5: results of the translocation analysis of GPS data at sites 1 and 4 on Mt. Everest.

error source	error (degrees centesimal)	notes
instrumental resolution	0.0001	
summit collimation	0.0020	(1)
refraction	0.0010	(2)
deflection of the vertical	0.0030	(3)
r.s.s.	0.0037	
network geometry error $\leq$ 10 m		(4)

notes:
(1) typical r.ms. of observations through a session
(2) 10% the maximum estimated correction for our data
(3) inferred tentatively from EDM-GPS comparison on Mt. Everest
(4) estimated for the weakest baseline

Table 6: error budget of the ground survey

Pseudo range resolution	wavelength (m)	1% of wavelength (m)
C/A code	300	3
Model Errors		
source	typical size (m)	
satellite orbit	20	
clock	10	
ionospheric propag.	30	
tropospheric propag.	10	
receiver multipath	10	
GDOP	15	
r.s.s.	42.7	

Table 7: error budget of GPS point positioning measurements with C/A code at the L1 frequency

Appendix

The atmospherics refraction index as a function of height.

We introduce the group refractivity N related to the index of refraction n by

$$N = 10^6 (n - 1)$$

The conventionally accepted value for N is

$$N = 80.343 f(\lambda) \frac{P}{T} 11.3 \frac{e}{T}$$

with

$f(\lambda) = 0.9650 + 0.0164/Y^2 + 0.000228/\lambda^4$
λ = wavelenght in micron (assume $\lambda = 0.6\mu m$)
P = total air pressure in millibar
e = partial pressure of water vapour in millibar
T = temperature in degree Kelvin
The partial pressure "e" is related to the observed humidity $H\%$ by

$$e = 0.0611 H\% \, 10^{\frac{7.5\,(T - 273.15)}{237.3 + (T - 273.15)}} \text{ millibar}$$

(Marini and Murray, 1973).
As to the dependence of pressure and temperature on heigth we assumes (Rostagni, 1957)

$$P(h) = P(h^\circ)\exp[-(h - h^\circ)/(<T>29.4)]$$
$$T(h) = T(h^\circ) - 0.0065(h - h^\circ)$$

where h° is the height of the reference site and

$$<T>=1/2[T(h^\circ) + T(h1)].$$

The linear dependence of temperature on heigth above assumes that $h1$ is smaller than the heigth of tropopause, wich, in fact, applies to our case.

Fig. 1

The geodetic network for the measurement of Mt. Everest. (From a Chinese map.)

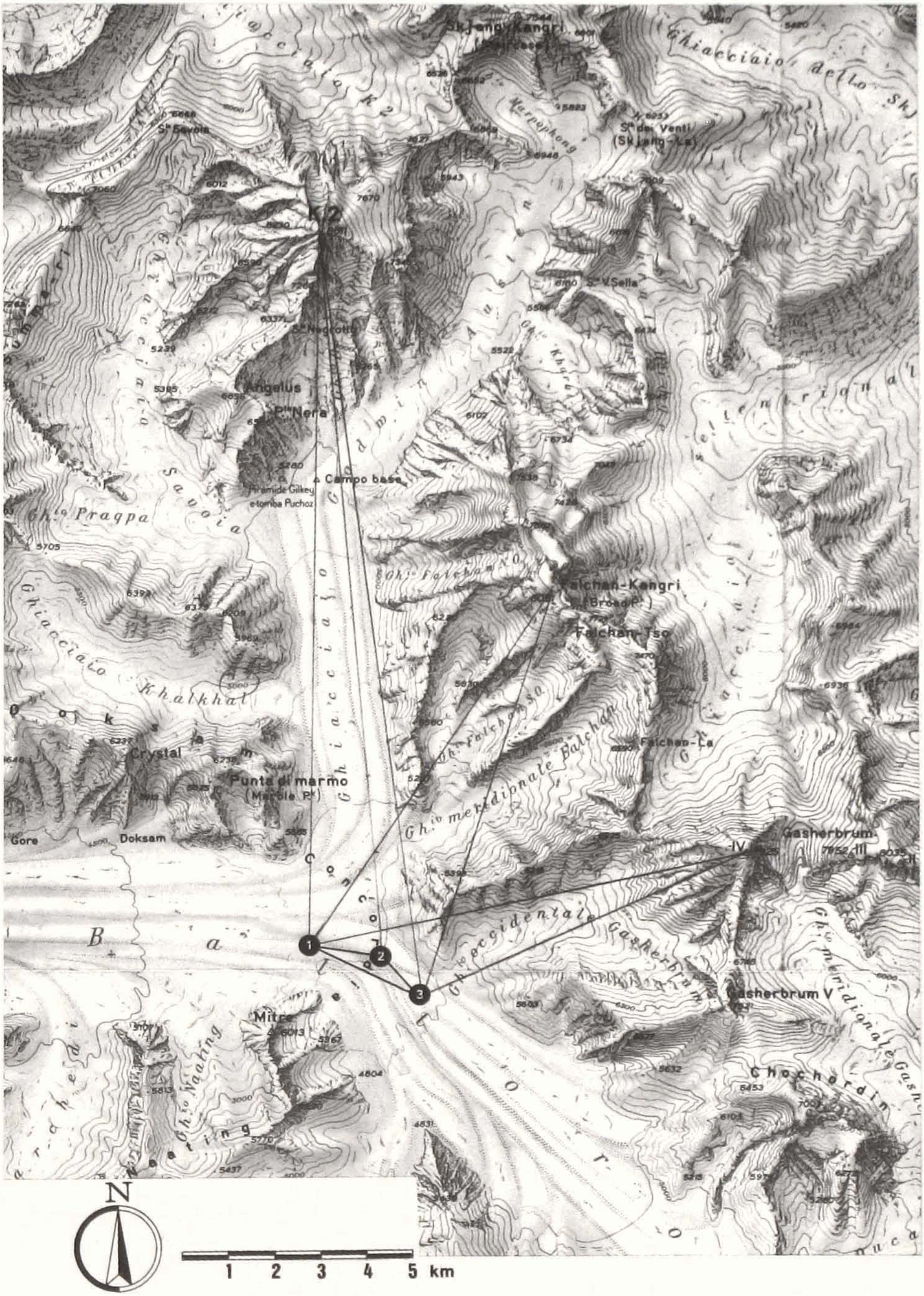
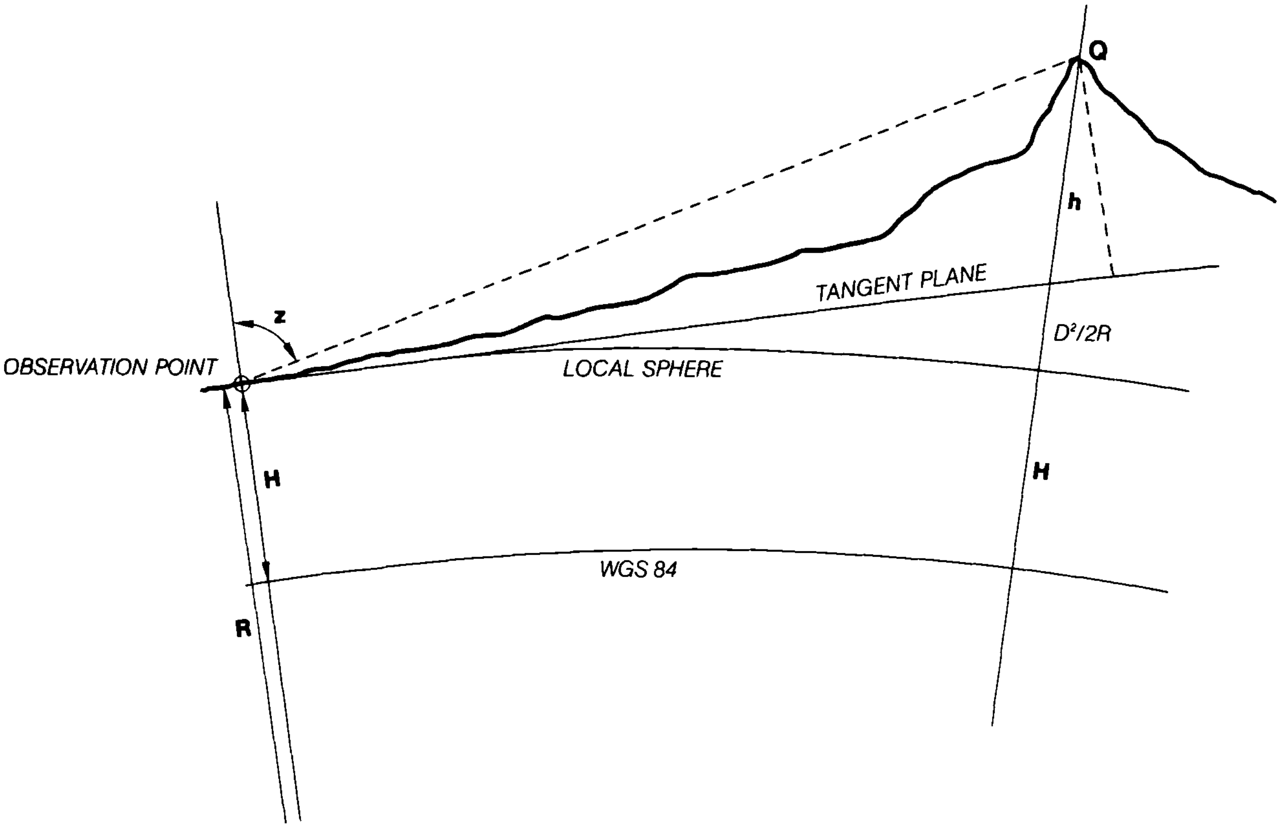


Fig. 2
The geodetic network for the measurement of K², Falchan Kangri and Gasherbrum IV (adapted, with permission, from the 1:100,000 map of the Baltoro Glacier of the Italian Expedition to K², 1954, by the Istituto Geografico Militare Italiano).



Q *summit* $= h + D^2/2R + H$

h: *height of summit relative to the plane tangent to the local sphere at the observation point*

D: *topographic distance from the observation point to the summit*

H: *GPS height of the observation point relative to the WGS 84 ellipsoid*

R: *radius of the local sphere of the observation point*

Fig. 3

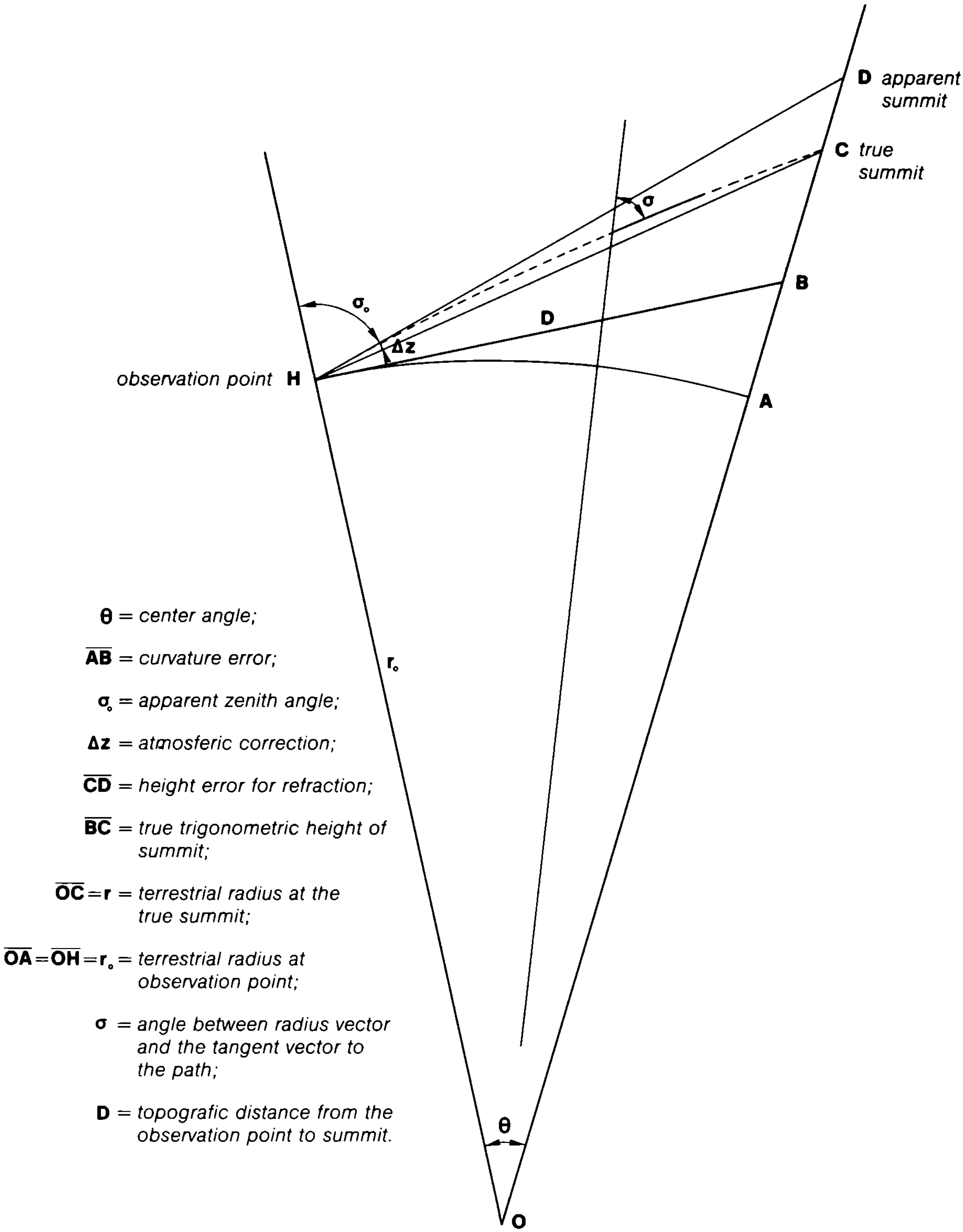

D apparent summit
C true summit
σ
σ_o
Δz
D
B
observation point H
A
r_o
θ
O
θ = center angle;
$\overline{AB}$ = curvature error;
σ_o = apparent zenith angle;
Δz = atmosferic correction;
$\overline{CD}$ = height error for refraction;
$\overline{BC}$ = true trigonometric height of summit;
$\overline{OC}$ = r = terrestrial radius at the true summit;
$\overline{OA}$ = $\overline{OH}$ = r_o = terrestrial radius at observation point;
σ = angle between radius vector and the tangent vector to the path;
D = topografic distance from the observation point to summit.

Fig. 4

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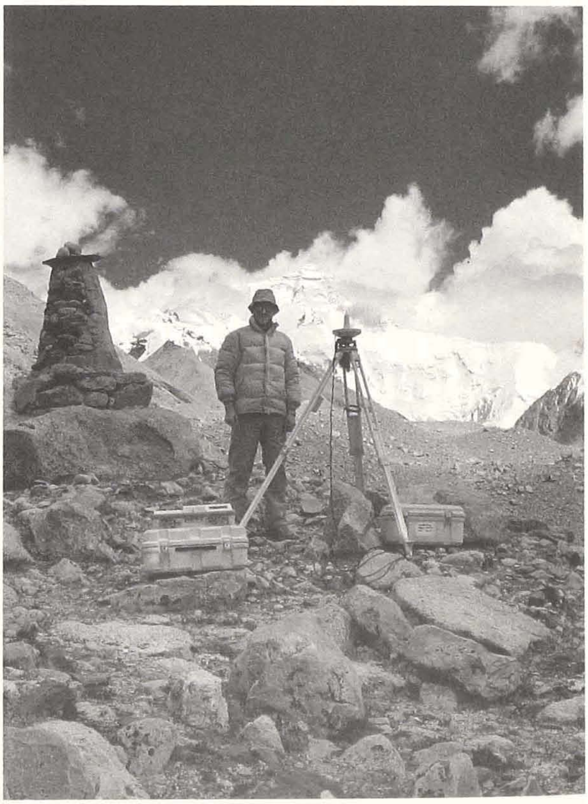
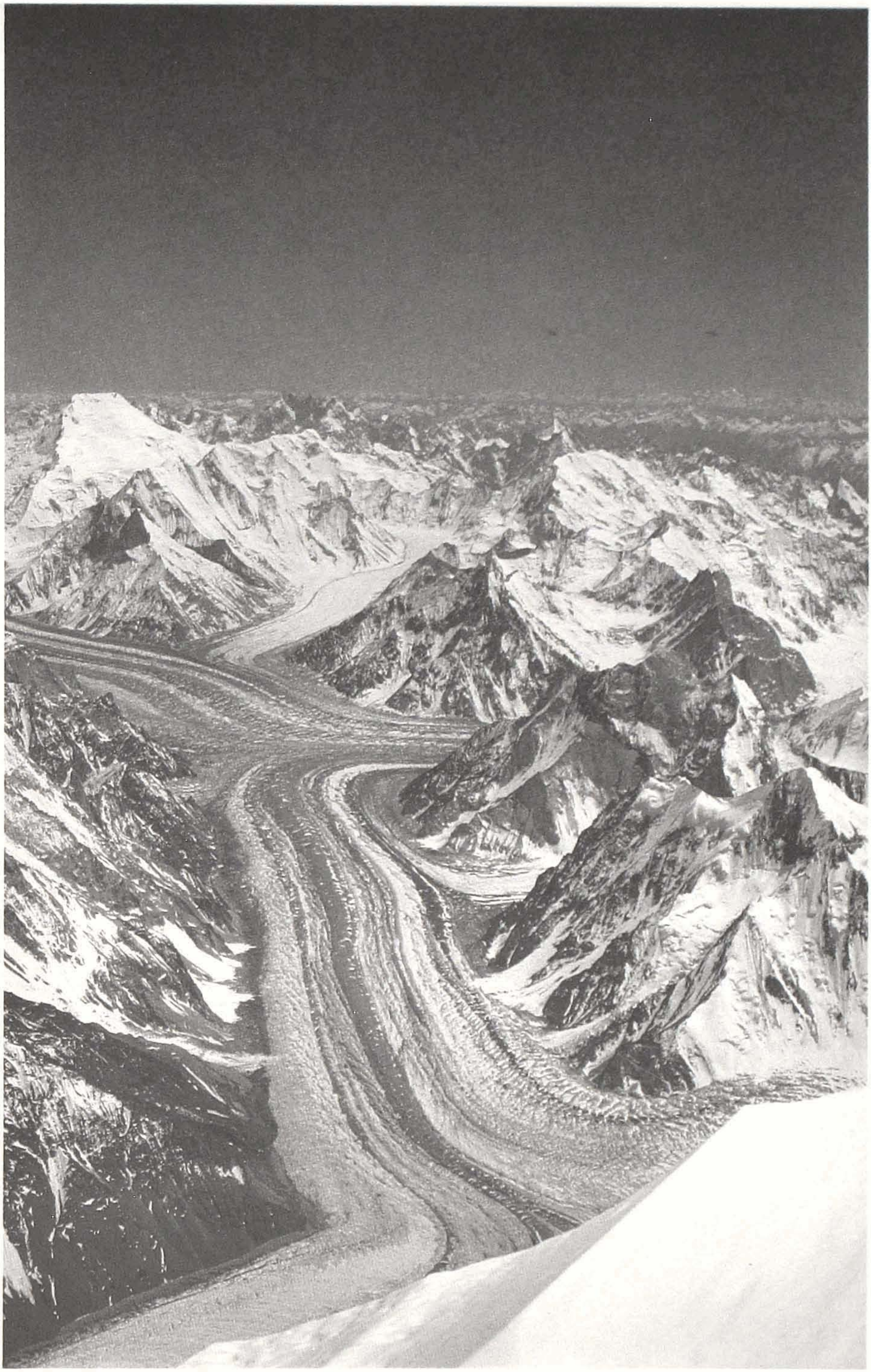
Rongbuck and Baltoro - August 1987

Everest seen from Rongbuk. It was here that the Ev-K²-CNR expedition had its base camp.



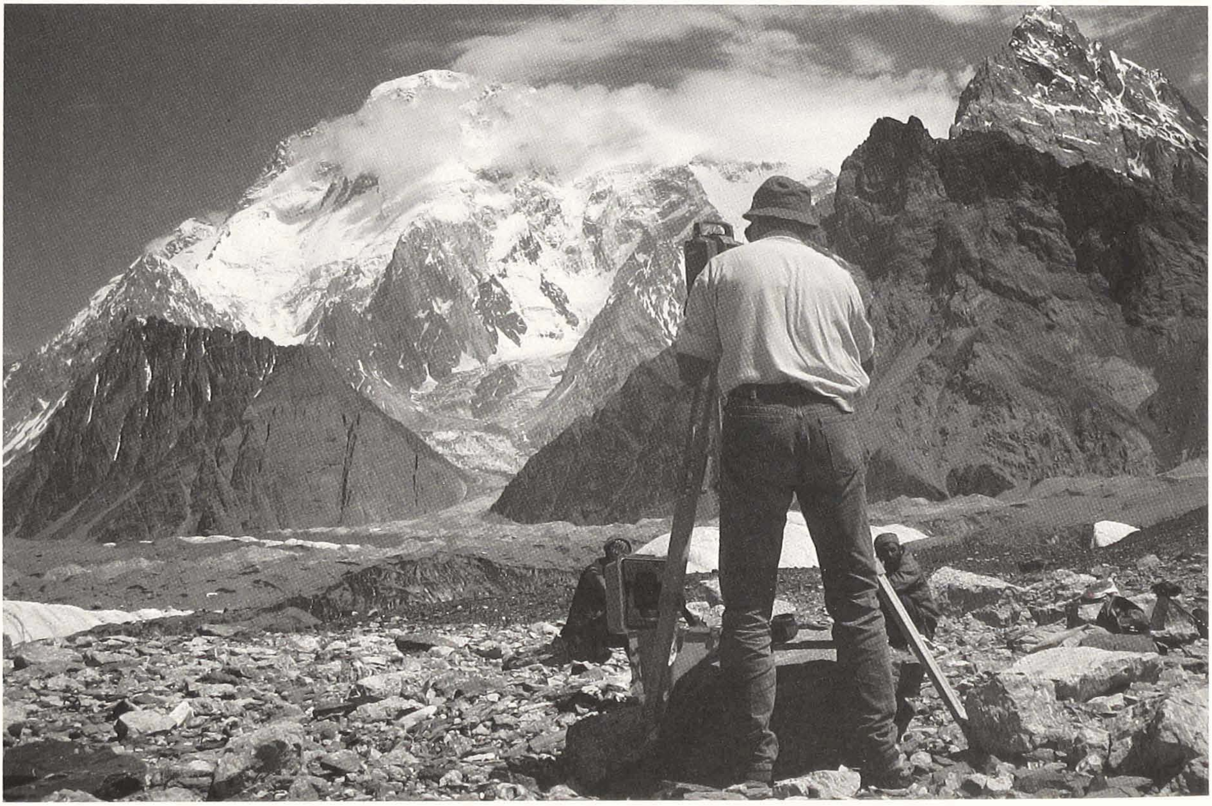
Taking measurements of Everest. Alessandro Caporali next to the stupa, taken as datum point no. 4 in the measurement process.



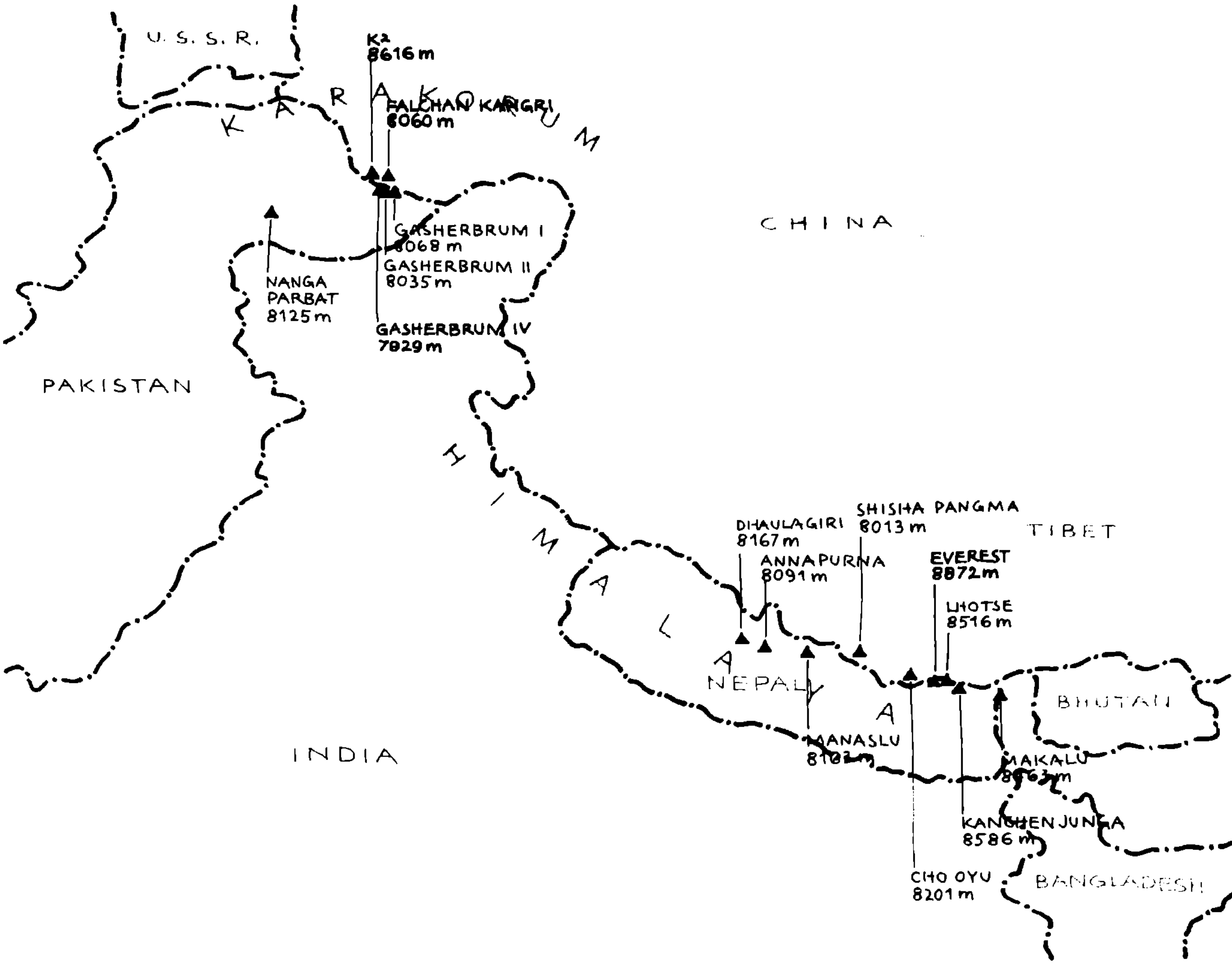
From the peak of K^2 looking south. In the centre, the Concordia Circus, convergence of the Baltoro glaciers, the base used for measurement.

Alessandro Caporali, Claudio Pigato and the cameraman Kurt Diemberger. In the background Gasherbrum IV.

Falchan Kangri (Broad Peak).



Caporali, Lavarini, Pigato and a porter facing K^2.



The Himalaya and Karakorum ranges, with the fourteen 8000 m peaks and Gasherbrum IV. Measurement results are marked in bold type.